# "混合型偏微分方程暨相关领域的前言问题" 学术研讨会



华中师范大学数学与统计学学院 三峡大学三峡数学研究中心

武汉 宜昌

2018年10月26日至31日

### "混合型偏微分方程暨相关领域的前言问题"学术研讨会

#### 学术委员会

主席: 辛周平

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#### 承办单位

华中师范大学数学与统计学学院

三峡大学三峡数学研究中心

#### 资助单位

国家自然科学基金天元基金 华中师范大学数学与统计学学院 三峡大学三峡数学研究中心

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湖北省武汉市洪山区珞喻路152号华中师范大学数学与统计学学院 邮编: 430079

### 会议时间安排

- ♦ 学术讲座报到时间: 2018年10月26日(星期五下午2点)
- ♦ 学术会议报到时间: 2018年10月28日(星期天下午2点)
- ♦ 学术讲座报到地点: 武汉雄楚国际酒店,武汉市.
- ♦ 学术会议报到地点: 宜昌金德瑞国际酒店,宜昌市.
- ◆ 学术讲座: 2018年10月27日至28日学术讲座地点: 数学与统计学学院6401教室
- ◆ 学术会议: 2018年10月29日至31日学术会议地点: 三峡大学理科楼L2620
- ♦ 离会: 2018年11月1日
- ♦ 就餐地点: 桂苑宾馆(武汉);金德瑞国际酒店(宜昌)
- ♦会议联系人:

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# 会议日程摘要

	10月27日	10月28日	10月29日	10月30日	10月31日
	辛周平	徐超江	江	赵会江	
	(学术讲座)	(学术讲座)	松	李竞	
上			朱熹平	罗 涛	学
		茶歇			
	辛周平	徐超江	曹道民	唐春雷	交
午	(学术讲座)	(学术讲座)	王春朋	谢春景	流
			欧乾忠	周茂林	
中午	午餐及午休				
	汪徐家	辅导答疑	李海梁		
	(学术讲座)	学术交流	酒全森		
下			杜力力	自	学
	茶歇		茶歇	由	术
	汪徐家	离开武汉	刘兆理	讨	交
午	(学术讲座)	前往宜昌	丁时进	论	流
			黄耿耿		
晚上			晚餐	·	·

# 学术讲座日程

日期/时间	10月26日	10月27日	10月28日	
8:00-8:30		注册6401教室		
		跨音速流中的	动力学方程和边界	
8:30-10:00		一些问题	层方程的亚椭圆性	
		主讲人: 辛周平	主讲人:徐超江	
10:00-10:20		茶歇	茶歇	
		跨音速流中的	动力学方程和边界	
10:20-11:50		一些问题	层方程的亚椭圆性	
		主讲人: 辛周平	主讲人:徐超江	
11:50-14:30		午餐:桂苑宾馆	午餐: 桂苑宾馆	
		Topics in Optimal	辅导答疑,学术交流	
14:00-16:00		Transport	6401辛周平	
		主讲人: 汪徐家	6507汪徐家	
16:00-16:20		茶歇	6511徐超江	
	注册:	Topics in Optimal	部分专家	
16:20-17:50	6401教室	Transport 离开武汉去宜		
		主讲人:汪徐家		
18:00	晚餐	晚餐:桂苑宾馆	晚餐:桂苑宾馆	

日期: 10月29日

主席	时间	报告人,题目	地点	
彭双阶	8:10 8:40	开幕式及照相	L2620	
汪徐家	8:40 9:20	江松,北京应用物理与数学研究所 Magnetic inhibition effect on the Rayleigh-Taylor instability in non-resistive magnetohydrodynamics	L2620	
严树森	9:20 10:00	朱熹平,中山大学 On the Einstein Field Equation	L2620	
	10:00-	-10:20 茶歇		
杨健夫	10:20 10:50	曹道民,中国科学院数学与系统科学研究院 一个超临界变指数半线性椭圆方程的无穷多解	L2620	
王维克	10:50 11:20	王春朋, 吉林大学 Subsonic and sonic jet flows for given surrounding pressures from convergent nozzles	L2620	
杨孝平	11:20 11:50	欧乾忠, 广西师范大学 On the second boundary value problem for a class of fully nonlinear equations	L2620	
午餐(金德瑞国际酒店)				

# 学术报告安排表

## 日期:10月29日

主席	时间	报告人,题目	地点	
郭宗明	$14:30 \\ 15:00$	李海梁,首都师范大学 Spectrum and Behaviors of Vlasov-Poisson- Boltzmann equations	L2620	
	15:00 15:30	酒全森, 首都师范大学 Global weak solutions to 3D incompressible ax- isymmetric Euler equations without swirls	L2620	
郭玉霞	15:30 16:00	杜力力,四川大学 Compressible subsonic impinging jet flows	L2620	
	16:00-	-16:20 茶歇		
周焕松	16:20 16:50	刘兆理,首都师范大学 Vector solutions with prescribed component-wise nodes for a Schrödinger system	L2620	
	16:50 17:20	丁时进, 华南师范大学 Boundary Layer for 3D Plane Parallel Channel Flows of Nonhomogeneous Incompressible Navier- Stokes Equations	L2620	
郭玉劲	17:20 17:50	黄耿耿, 复旦大学 Uniqueness of least energy solutions for Monge- Ampère functional	L2620	
	晚餐(金德瑞国际酒店)			

# 学术报告安排表

日期: 10月30日

主席	时间	报告人,题目	地点	
陈化	8:30 9:00	赵会江,武汉大学 On the Vlasov-Maxwell-Boltzmann System	L2620	
	9:00 9:30	李竞,中科院 Global Existence of Weak Solutions to the Barotropic Compressible Navier-Stokes Flows with Degenerate Viscosities	L2620	
谭忠	$9:30 \\ 10:00$	罗涛, 香港城市大学 Some Results on Plasma/MHD Equations with Boundaries	L2620	
	10:00-	-10:20 茶歇		
谭忠	$10:20 \\ 10:50$	唐春雷,西南大学 Some results for Choquard equations	L2620	
李海刚	10:50 11:20	谢春景,上海交通大学 Subsonic flows in nozzles and asymptotic behavior for elliptic equations in cylindrical domains	L2620	
	$11:20 \\ 11:50$	周茂林,新英格兰大学 Logarithmic corrections in Fisher-KPP problems for the Porous Medium Equation	L2620	
午餐(金德瑞国际酒店)				

# 报告摘要

### 一个超临界变指数半线性椭圆方程的无穷多解

曹道民,中国科学院数学与系统科学研究院

报告人将介绍和合作者李双龙、刘忠原最近在研究超临界变指数半 线性椭圆方程所取得的一个结果.所研究的方程的区域为单位球,方程本 身也具有球对称形式,边值为带零Dirichlet边值,非线性是带超临界变指 数项.这类问题由于对应的变分泛函不再有紧性,因而研究起来很困难. 我们通过运用变分方法,先分别在球形区域和环形区域构造出球对称正 解,最后"组装"成单位球上具有给定零点个数的变号解.

# Boundary Layer for 3D Plane Parallel Channel Flows of Nonhomogeneous Incompressible Navier-Stokes Equations

## 丁时进, 华南师范大学

In this talk, we establish the mathematical validity of the Prandtl boundary layer theory for a class of nonlinear plane parallel ow of nonhomogeneous incompressible Navier-Stokes equations. The convergence for the density and velocity are shown under various Sobolev norms, including the physically important space-time uniform norm, as well as the  $L^{\infty}(H^1)$ norm. In addition, the higher-order asymptotic expansions are also considered. It is mentioned that the mathematical validity of the Prandtl boundary layer theory for nonlinear plane parallel ow is generalized to the nonhomogeneous case. This is a joint work with Zhilin Lin and Dongjuan Nin.

#### Compressible subsonic impinging jet flows

# 杜力力,四川大学

In this talk, we will introduce the recent progress on the compressible subsonic jet flows, including the mathematical theory on the steady compressible subsonic impinging flows, steady incompressible impinging jet and compressible subsonic impinging jet flows.

## Uniqueness of least energy solutions for Monge-Ampère functional

## 黄耿耿,复旦大学

In this talk, we discuss the minimizing problem of the following functional

$$\mathcal{E}[u] = \int_{\Omega} (-u) \det D^2 u dx, \quad \|u\|_{L^{q+1}(\Omega)} = 1$$
(1)

where  $u \in C^2(\overline{\Omega})$  is convex and u = 0 on  $\partial\Omega$ . The uniqueness of least energy solution of (1) is investigated. For n = 2, we prove the least energy solution of (1) is unique for  $2 < q < \infty$  provided it is locally uniformly convex. In particular, for  $q = +\infty$ , we show the uniqueness of the least energy solution of (1) and find its relation to Santalò point.

# Magnetic inhibition effect on the Rayleigh-Taylor instability in non-resistive magnetohydrodynamics

江松,北京应用物理与数学研究所

The Rayleigh-Taylor (RT) instability is well known as gravity-driven instability in fluids when a heavy fluid is on top of a light one. It appears in a wide range of applications in science and technology, such as in inertia confinement fusion, Tokamak, supernova explosions. In this talk, mathematical analysis of the magnetic RT instability in both incompressible and compressible fluids will be presented, in particular, effects of (impressed) magnetic fields upon the growth of the RT instability will be discussed and analyzed quantitatively. We shall show that a sufficiently strong (impressed) magnetic field can inhibit the RT instability; otherwise, instability will still occur in the sense that solutions do not continuously depend on initial data. Moreover, we shall give an explanation of physical mechanism for the magnetic inhibition phenomenon based on mathematical analysis.

## Global weak solutions to 3D incompressible axisymmetric Euler equations without swirls

酒全森,首都师范大学

In this talk, we present recent results on the global existence of a classical weak solution to the three dimensional incompressible axisymmetric Euler equations without swirl if the initial vorticity  $w_0^{\theta}$  satisfies that  $\frac{w_0^{\theta}}{r} \in L^1 \cap L^p(\mathbb{R}^3)$  for some p > 1. The result improves ones known before and is based on delicate estimates via Biot-Savart law.

#### Spectrum and Behaviors of Vlasov-Poisson-Boltzmann equations

李海梁,首都师范大学

The Vlasov-Poisson-Boltzmann equations can be used to model the transport of charged particles (one carrier or two carriers) under the influence of electrostatic potential force. It is not well understood yet how the electrostatic potential force and/or the mutual interaction between charged particles of different type shall affect the spectrum structures and the asymptotical behaviors of global solutions. In this talk, we present

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the recent progress on the analysis on the spectrum structure and optimal pointwise space-time behaviors of three dimentional Vlasov-Poisson-Boltzmann equations, and the nonlinear stability of planar wave pattern for the bipolar Vlasov-Poisson-Boltzmann equations, such as including shock profile, rarefaction wave and contact discontinuity. These works are joint with Tong Yang, Ming-Ying Zhong, Yi Wang, and Teng Wang.

# Global Existence of Weak Solutions to the Barotropic Compressible Navier-Stokes Flows with Degenerate Viscosities

# 李竞,中科院

We consider the existence of global weak solutions to the barotropic compressible Navier-Stokes equations with degenerate viscosity coefficients. We construct suitable approximate system which has smooth solutions satisfying the energy inequality, the BD entropy one, and the Mellet-Vasseur type estimate. Then, after adapting the compactness results due to Bresch-Desjardins (2002, 2003) and Mellet-Vasseur (2007), we obtain the global existence of weak solutions to the barotropic compressible Navier-Stokes equations with degenerate viscosity coefficients in two or three dimensional periodic domains or whole space for large initial data. This, in particular, solved an open problem proposed by Lions (1998). This is a joint work with Prof. Zhouping Xin.

## Some Results on Plasma/MHD Equations with Boundaries

## 罗涛,香港城市大学

In this talk, I shall discuss some problems for plasma and MHD equations with boundaries. For viscous plasma equations with fixed boundaries, I will discuss the result joint work with Qiangchang Ju and Xin Xu on the singular limit. The recent result joint with Chengchun Hao on the Instability of a free boundary problem of MHD will be also discussed.

## Vector solutions with prescribed component-wise nodes for a Schrödinger system

刘兆理,首都师范大学

For the Schrödinger system

$$\begin{cases} -\Delta u_j + \lambda_j u_j = \sum_{i=1}^k \beta_{ij} u_i^2 u_j & \text{in } \mathbb{R}^N, \\ u_j(x) \to 0 & \text{as } |x| \to \infty, \quad j = 1, \cdots, k, \end{cases}$$

where  $k \geq 2$  and N = 2, 3, we prove that for any  $\lambda_j > 0$  and  $\beta_{jj} > 0$ and any positive integers  $p_j$ ,  $j = 1, 2, \dots, k$ , there exists b > 0 such that if  $\beta_{ij} = \beta_{ji} \leq b$  for all  $i \neq j$  then there exists a radial solution  $(u_1, u_2, \dots, u_k)$  with  $u_j$  having exactly  $p_j - 1$  zeroes. Moreover, there exists a positive constant  $C_0$  such that if  $\beta_{ij} = \beta_{ji} \leq b$   $(i \neq j)$  then any solution obtained satisfies

$$\sum_{i,j=1}^k |\beta_{ij}| \int_{\mathbb{R}^N} u_i^2 u_j^2 \le C_0.$$

Therefore, the solutions exhibit a trend of phase separations as  $\beta_{ij} \to -\infty$  for  $i \neq j$ .

# On the second boundary value problem for a class of fully nonlinear equations

In this talk, we discuss the existence of convex solution to a class of fully nonlinear elliptic equations with second boundary condition on uniformly convex domains in  $\mathbb{R}^n$ , and then applied it to solve a boundary value problem for minimal Lagrangian graphs in the pseudo-Euclidean space  $\mathbb{R}_n^{2n}$ .

#### Some results for Choquard equations

## 唐春雷, 西南大学

In this talk, the main aim is to present our results around the mathematical study of Choquard equations and its variants. The results will be split into to three parts, that is, Ground state solution, Change-sign solution and multiple solutions. We also report the physical background and related variants and extensions of Choquard equations in this talk.

# Subsonic and sonic jet flows for given surrounding pressures from convergent nozzles

王春朋, 吉林大学

This talk concerns the compressible subsonic and sonic jet flows for a given surrounding pressure from a two-dimensional finitely long convergent nozzle with straight solid wall. For a given surrounding pressure and a given incoming mass flux, we seek a subsonic or sonic jet flow with the given incoming mass flux such that the flow velocity at the inlet is along the normal direction, the flow satisfies the slip condition at the wall, and the pressure of the flow at the free boundary coincides with the given surrounding pressure. The well-posedness is shown and the properties of the flow are investigated.

# Subsonic flows in nozzles and asymptotic behavior for elliptic equations in cylindrical domains

谢春景,上海交通大学

In this talk we will address the wellposedness theory and asymptotic behavior for subsonic flows in nozzles. The results also share the feature for the solutions of general elliptic equations in cylindrical domains. The basic techniques are energy estimate and Harnack estimate.

## On the Vlasov-Maxwell-Boltzmann System

## 赵会江,武汉大学

In this talk, we will review some of our recent results on the construction of global solutions of the Cauchy problem of the Vlasov-Maxwell-Boltzmann system.

# Logarithmic corrections in Fisher-KPP problems for the Porous Medium Equation

周茂林,新英格兰大学

We consider the large time behaviour of solutions to the porous medium equation with a Fisher-KPP type reaction term and give out a sharp estimate on the propagation of the solution. It is a joint work with Yihong Du in University of New England and Fernando Quirós in Universidad Autónoma de Madrid.

## On the Einstein Field Equation

朱熹平,中山大学

The well-known weak cosmic censorship conjecture states that for generic asymptotically flat initial data, the future null infinity is always complete. In this talk, we will show that the future null infinity is an open set. Since completeness is equivalent to openness and closeness, our result could be considered as the first step towards the weak cosmic censorship. This is a joint work with Junbin Li.